Hillside Calculation Policy

At Hillside, we use the CPA approach within our maths lessons (CPA – Concrete/ Pictorial/ Abstract). The CPA approach builds on children's existing knowledge by introducing abstract concepts in a concrete and tangible way. It involves moving from concrete materials, to pictorial representations, to abstract symbols and problems. The different stages are defined in detail below.

Concrete step of CPA

Concrete is the "doing" stage. During this stage, our children use concrete objects to model problems. Unlike traditional maths teaching methods where teachers demonstrate how to solve a problem, the CPA approach brings concepts to life by allowing children to experience and handle physical (concrete) objects.

For example, if a problem involves adding pieces of fruit, children can first handle actual fruit. From there, they can progress to handling abstract counters or cubes which represent the fruit.

Pictorial step of CPA

Pictorial is the "seeing" stage. Here, visual representations of concrete objects are used to model problems. This stage encourages our children to make a mental connection between the physical object they just handled and the abstract pictures, diagrams or models that represent the objects from the problem.

Building or drawing a model makes it easier for children to grasp difficult abstract concepts (for example, fractions). Simply put, it helps children visualise abstract problems and make them more accessible.

Abstract step of CPA

Abstract is the "symbolic" stage, where children use abstract symbols to model problems. Our children will not progress to this stage until they have demonstrated that they have a solid understanding of the concrete and pictorial stages of the problem. The abstract stage involves the teacher introducing abstract concepts (for example, mathematical symbols). Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols (for example, +, -, x, /) to indicate addition, multiplication or division.

It is important to recognise that the model we use is progressive. By the end of KS1, our children need to be able to go beyond the use of concrete equipment to access learning using either pictorial representations or abstract understanding. What is important, therefore, is that all learners, however young, can see the connections between each representation.

We also use the philosophy of:

- fluency

- reasoning

- problem-solving

Mathematical fluency skills helps children think faster and more clearly, it is the ability to accurately recall mathematical concepts and facts. Having these crucial skills gives children the energy, attention and focus to tackle complex problem-solving and reasoning questions. Mathematical reasoning is the process of applying logical thinking to a situation to derive the correct problem solving strategy for a given question, and using this method to develop and describe a solution. It is the bridge between fluency and problem solving. It allows our children to use the former to accurately carry out the latter. At Hillside, we do not see problem solving as something that some pupils can do and some cannot. We believe very child is born with an innate level of problem-
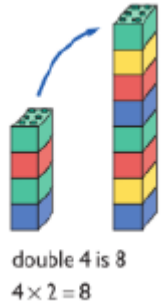
solving ability. Our ability to successfully problem solve requires us to have a deep understanding of content and fluency of facts and mathematical procedures. Critical thinking processes such as reasoning and problem solving—are intimately intertwined with factual knowledge that is stored in long-term memory. Embedding information to the long term memory underpins our curriculum offer and we work hard to embed metacognition into our learning and everyday language to ensure children are thinking about their learning and how they can retain the information they have learned.

The aim is that when children leave Hillside they:

- Have a secure knowledge of number facts and a good understanding of the four calculation operations (addition, subtraction, multiplication and division)
- Make use of jottings, diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads
- Have an efficient, reliable, written method of calculation for each operation that they are able to apply with confidence when they are unable to perform a calculation mentally

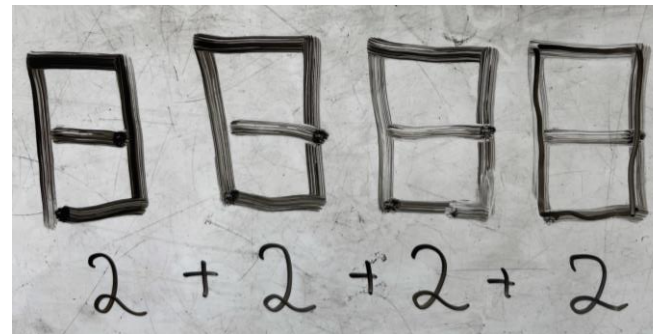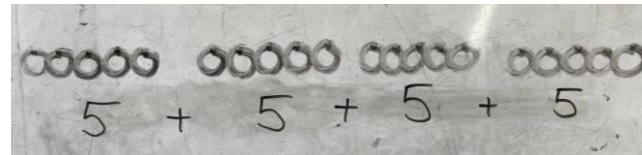<u>Progression in Calculations</u>

<u>Multiplication</u>

Objective and Strategies	Concrete	Pictorial	Abstract
<u>Year 1 Multiplication</u> Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.			
Doubling	double 4 is 8 $4 \times 2 = 8$ Use practical activities using manipulatives including cubes and numicon to show how to double a number.	Draw pictures to show how to double a number. Double 4 is 8	4+ 4 = 8

Repeated addition	 Count in multiples supported by concrete objects in equal groups.	Use a number line or pictures to continue support in counting in multiples. 5 + 5 + 5 + 5 2 + 2 + 2 + 2	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30 Write addition sentences to describe objects and pictures. 2 + 2 + 2 + 2 + 2 = 10
Making equal groups and counting the total.	Use manipulatives to create equal groups. 4 × 2	Draw and make representations. □ × □ = 8	4 × 2 = 8
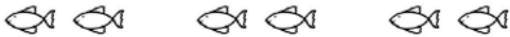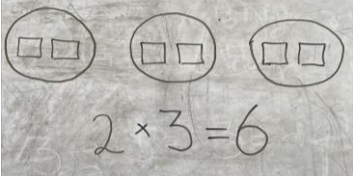
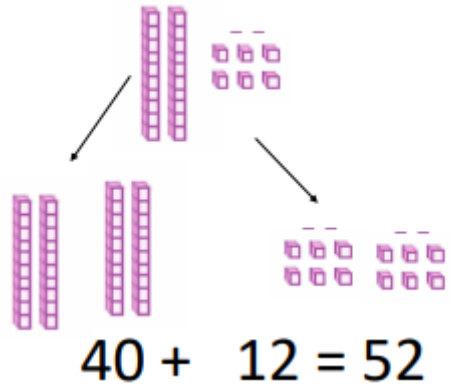|		Draw 🐟 to show 2 x 3 = 6 🐟 🐟 🐟 🐟 🐟 🐟	2×3=6
Understanding arrays	Use objects laid out in arrays to find the answers to 4 lots of 6… Or 4 lots of 2…	Draw representations of arrays to show understanding. Draw arrays in different orientations to show communtativity. 4 × 2 = 8 2 × 4 = 8 2 × 4 = 8 4 × 2 = 8	Use arrays to write multiplication sentences. 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Objective and Strategies	Concrete	Pictorial	Abstract
Year 2 Multiplication			

Recall and use **multiplication and division facts** for the 2, 5 and 10 multiplication tables, including recognising **odd and even numbers**

Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts
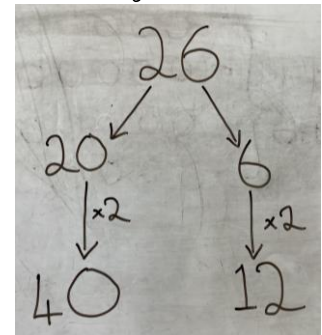
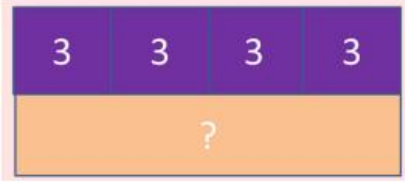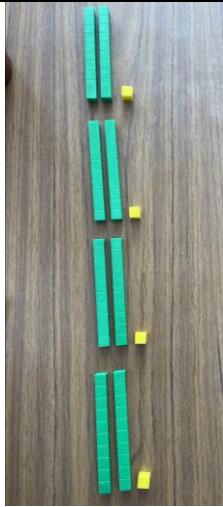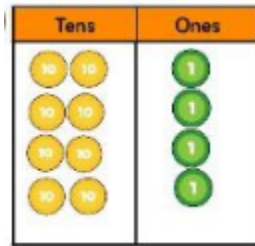| Doubling	Model doubling using dienes and PV counters. E..g double 26...	Draw pictures and representations to show how to double numbers.	Partition a number and then double each part before combining it back together.
		40 + 12 = 52	
Counting in multiples of 2, 3, 5 and 10 from 0.	Count the groups as children are skip counting. Children may use their fingers as they	Number lines, counting sticks and bar models should be used to show representations of counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0,2,4,6,8,10 0,3,6,9,12,15
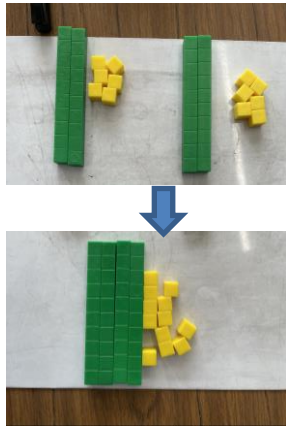
skip count.

$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$

0,5,10,15,20,25,30

$4 \times 3 -$ ___

Appropriate strategies to be used with consistent practice.

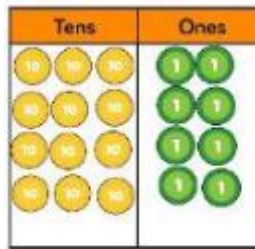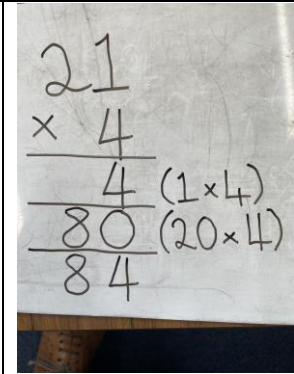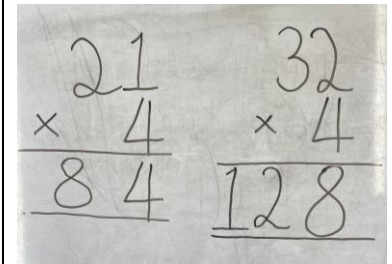| Objective and Strategies	Concrete	Pictorial	Abstract
Year 3 Multiplication Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods Progress to formal written methods calculations as above			
To multiply 2 digit numbers by 1 digit numbers.	Children can continue to be supported by Dienes and place value counters at this stage of multiplication.	Children can represent their work with place value counter or drawn representation in a way that they understand. They can draw the counters using colours to show different amounts or just use circles in the different columns to show their thinking.	Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

8

21 × 4

32 × 4

Tens | Ones

21 × 4 =

Tens | Ones

32 × 4 =

$$\begin{array}{r} 21 \\ \times\ 4 \\ \hline 4\ (1\times4) \\ 80\ (20\times4) \\ \hline 84 \end{array}$$

Formal written method of short multiplication, 2 digit by 1 digit.

$$\begin{array}{r} 21 \\ \times\ 4 \\ \hline 84 \end{array} \qquad \begin{array}{r} 32 \\ \times\ 4 \\ \hline 128 \end{array}$$

| Objective and Strategies	Concrete	Pictorial	Abstract

Year 4 Multiplication

Recall multiplication and division facts for multiplication tables up to 12 × 12

Multiply two-digit and three-digit numbers by a one-digit number using formal written layout
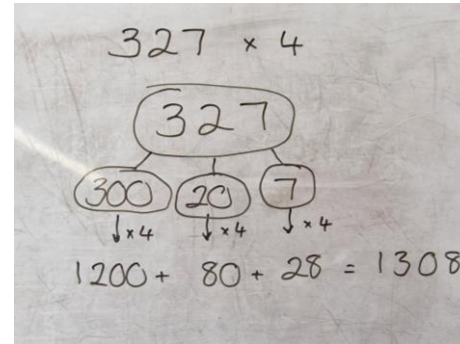
| Column multiplication | Children can continue to be supported by PV counters at this stage of multiplication. This initially done where there is no regrouping.

327 × 4 =
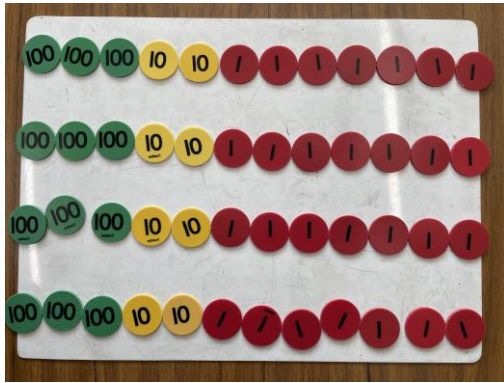
 | Part whole models can be useful for allowing children to understand the role of partitioning in multiplication.

It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below. | Begin with long multiplication to show the steps.
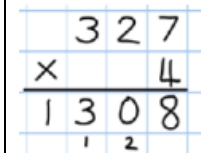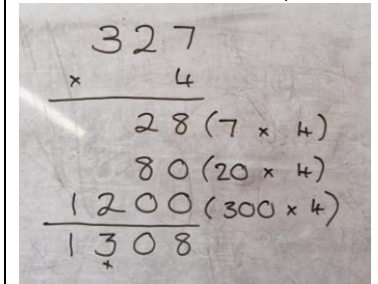
 This may lead to a compact method. |

Objective and Strategies	Concrete	Pictorial	Abstract

Year 5 Multiplication

Multiply and divide numbers mentally drawing upon known facts

Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes

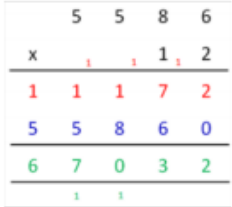| Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.			
Column multiplication Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers	As above- see Year 3 and Year 4	As above- see Year 3 and Year 4	Compact short multiplication of a four digit by a 1 digit number. Long multiplication of a 2, 3 or 4 digit number by a 2 digit number.

Objective and Strategies	Concrete	Pictorial	Abstract
Year 6 Multiplication			
Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication Use their knowledge of the order of operations to carry out calculations involving the four operations Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy			
Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication	See above	See above	Estimate by calculating prior to the written strategy. 5586 x 12 ⟶ 6000 x 10 ⟶ 60,000 Compact column addition to add decimal numbers with up to 2 places

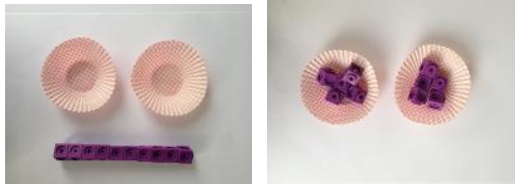| Multiplying decimals up to 2 decimal places by a single digit.	3.14 × 3		Remind the children that the single digit belongs in the ones column. Line up the decimal points in the question and the answer.
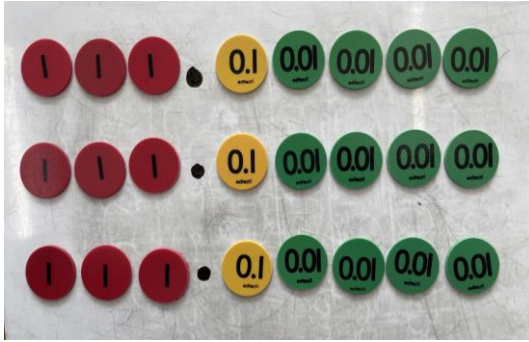
Division

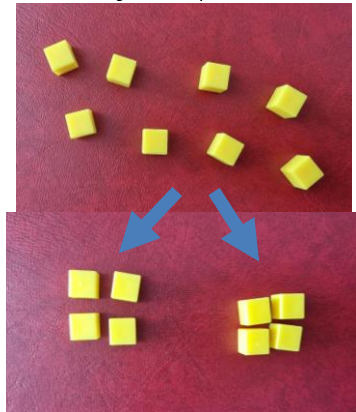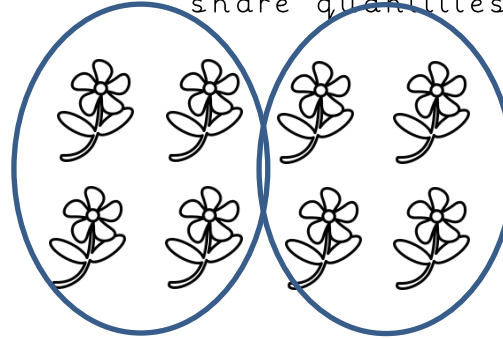| Objective and Strategies	Concrete	Pictorial	Abstract
Year 1 Division			
Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.			

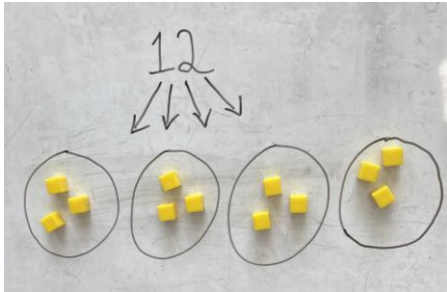| Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	I have 8 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.	Share 8 flowers between two people. 8 ÷ 2 = 4

Year 2 Division

Recall and use **multiplication and division facts** for the 2, 5 and 10 multiplication tables, including recognising **odd and even numbers**
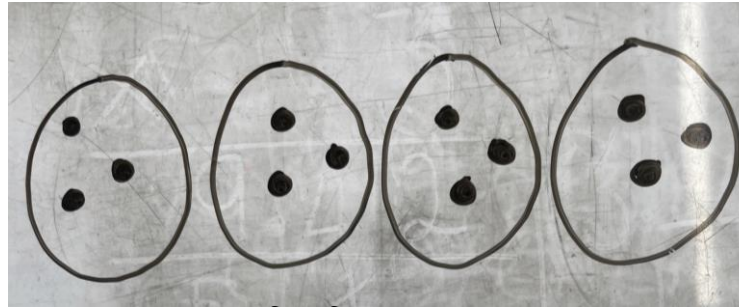
Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

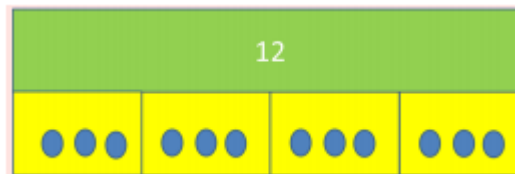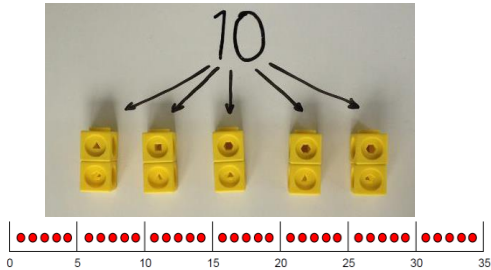|			
Division as sharing	I have twelve cubes, can you share them equally in four groups. 	Children use pictures or shapes to share quantities. Draw the groups first, then share equally between them. Children use bar modelling to show and support understanding 12 ÷ 4	12 ÷ 4 = 3

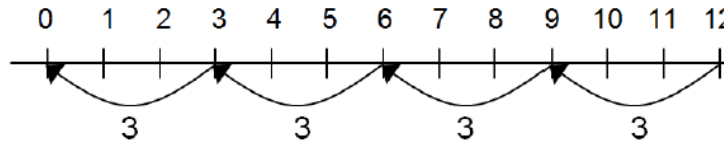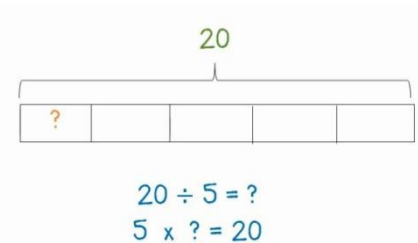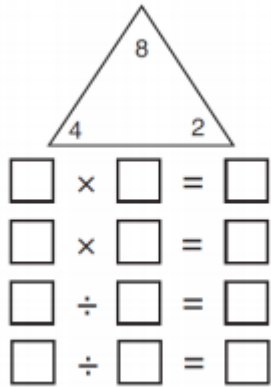|			
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. 96 ÷ 3 = 32 	Use a number line to show jumps in groups. The number of jumps equals the number of groups. Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 20 ÷ 5 = ? 5 x ? = 20	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

| Using the inverse

This should be taught alongside division, so children learn how they work alongside each other. | | □ × □ = □
□ × □ = □
□ ÷ □ = □
□ ÷ □ = □ | 2 x 4 = 8
4 x 2 = 8
8 ÷ 2 = 4
8 ÷ 4 = 2
8 = 2 x 4
8 = 4 x 2
2 = 8 ÷ 4
4 = 8 ÷ 2
Show all 8 related fact family sentences. |

Objective and Strategies	Concrete	Pictorial	Abstract

Year 3 Division

Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods
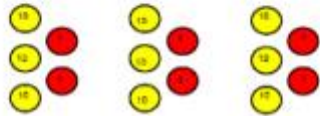
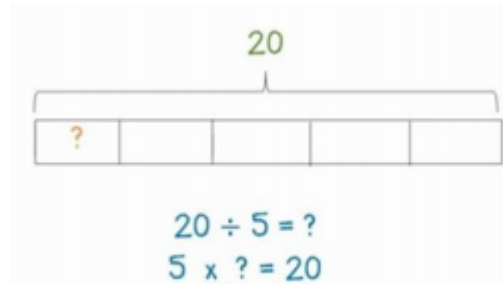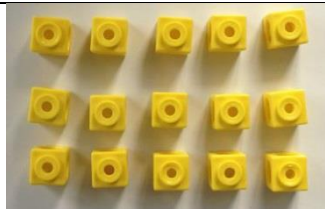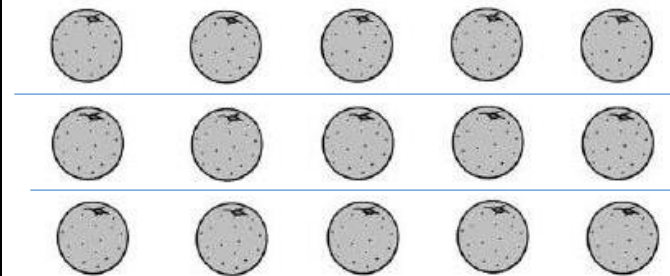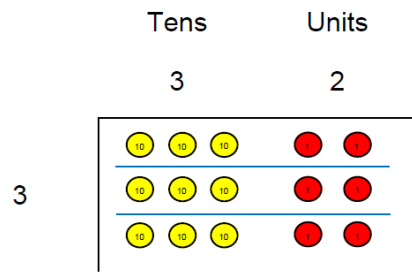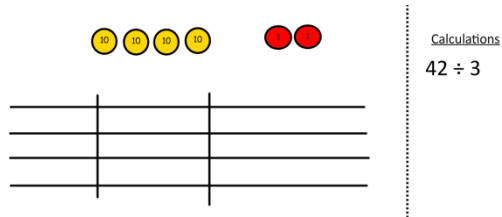| Division as grouping	Use cubes, counters, objects or place value 24 divided into groups of 6 = 4 $96 \div 3 = 32$ counters to aid understanding.	Continue to use bar modelling to aid solving division problems. 20 $20 \div 5 = ?$ $5 \times ? = 20$	How many groups of 6 in 24? $24 \div 6 = 4$
Division within arrays	Link division to multiplication by creating an array and thinking about the number	Draw an array and use lines to split the array into groups to make	Find the inverse of multiplication and division sentences by creating four linking number sentences. $7 \times 4 = 28$

	sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15 Provide children with stem sentences to articulate their understanding of division. Complete the stem sentences. I have ___ cubes altogether. There are ___ in each group. There are ___ groups.	multiplication and division sentences.	4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Objective and Strategies	Concrete	Pictorial	Abstract
Year 4 Division Recall multiplication and division facts for multiplication tables up to 12 × 12 Recognise and use factor pairs and commutativity in mental calculations Divide a three-digit number by a one-digit number Estimate and use inverse operations to check answers to a calculation			

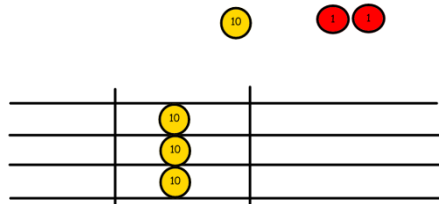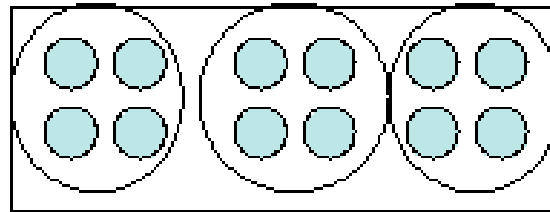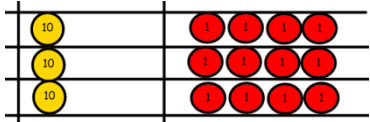| Divide at least 3 digit numbers by a 1 digit. Short division	Use place value counters to divide using the bus stop method alongside 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups. Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.

Tens   Units

3   2

3

Calculations
42 ÷ 3

218
    3
4 ⟌ 8 7 2

We exchange this ten for ten ones and then share the ones equally among the groups.

We look how much is in 1 group so the answer is 14.

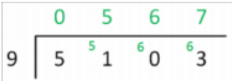| Objective and Strategies	Concrete	Pictorial	Abstract

Year 5 Division

Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes

Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.

Objective and Strategies	Concrete	Pictorial	Abstract
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context	As above	As above	Short division of a four-digit number by a one-digit number, using remainders when necessary. 0 5 6 7 9] 5 ⁵1 ⁶0 ⁶3 1 6 9 4 r2 5] 8 ³4 ⁴7 ²2

Year 6 Division

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to context

Use their knowledge of the order of operations to carry out calculations involving the four operations

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context	As above	As Above	The short and long division method will be used in Year 6 to divide 4 digit numbers by a 2 digit number. $$\begin{array}{r} 1\ 4\ .\ 6 \\ \hline 3\ 5\ |\ 5\ 1\ 1\ .\ 0 \end{array}$$ Children should be taught how to display the remainder as (the remainder can be interpreted as $\frac{12}{15} \rightarrow \frac{4}{5}$ or 0.8)

			decimal and a fraction.