

Hillside Calculation Policy



At Hillside, we use the CPA approach within our maths lessons (CPA - Concrete/ Pictorial/ Abstract). The CPA approach builds on children's existing knowledge by introducing abstract concepts in a concrete and tangible way. It involves moving from concrete materials, to pictorial representations, to abstract symbols and problems. The different stages are defined in detail below.

Concrete step of CPA

Concrete is the "doing" stage. During this stage, our children use concrete objects to model problems. Unlike traditional maths teaching methods where teachers demonstrate how to solve a problem, the CPA approach brings concepts to life by allowing children to experience and handle physical (concrete) objects.

For example, if a problem involves adding pieces of fruit, children can first handle actual fruit. From there, they can progress to handling manipulatives such as counters or cubes which represent the fruit.

Pictorial step of CPA

Pictorial is the "seeing" stage. Here, visual representations of concrete objects are used to model problems. This stage encourages our children to make a mental connection between the physical object they just handled and the more abstract pictures, diagrams or models that represent the objects from the problem.

Building or drawing a model makes it easier for children to grasp difficult abstract concepts (for example, fractions). Simply put, it helps children visualise abstract problems and make them more accessible.

Abstract step of CPA

Abstract is the "symbolic" stage, where children use abstract symbols to model problems. Often, our children will not progress to this stage until they have demonstrated that they have a solid understanding of the concrete and pictorial stages of the problem. The abstract stage involves the teacher

introducing abstract concepts (for example, mathematical symbols and numerals). Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols (for example, +, -, x, /) to indicate addition, multiplication or division.

It is important to recognise that the model we use is progressive. By the end of KS1, our children need to be able to go beyond the use of concrete equipment to access learning using either pictorial representations or abstract understanding. What is important, therefore, is that all learners, however young, can see the connections between each representation.

We also use the philosophy of:

- fluency
- reasoning
- problem-solving

Mathematical fluency skills helps children think faster and more clearly, it is the ability to accurately recall mathematical concepts and facts. Having these crucial skills gives children the energy, attention and focus to tackle complex problem-solving and reasoning questions. Mathematical reasoning is the process of applying logical thinking to a situation to derive the correct problem solving strategy for a given question, and using this method to develop and describe a solution. It is the bridge between fluency and problem solving. It allows our children to use the former to accurately carry out the latter. At Hillside, we do not see problem solving as something that some pupils can do and some cannot. We believe every child is born with an innate level of problem-solving ability. Our ability to successfully problem solve requires us to have a deep understanding of content and fluency of facts and mathematical procedures. Critical thinking processes such as reasoning and problem solving – are intimately intertwined with factual knowledge that is stored in long-term memory. Embedding information to the long term memory underpins our curriculum offer and we work hard to embed metacognition into our learning and everyday language to ensure children are thinking about their learning and how they can retain the information they have learned.

The aim is that when children leave Hillside they:

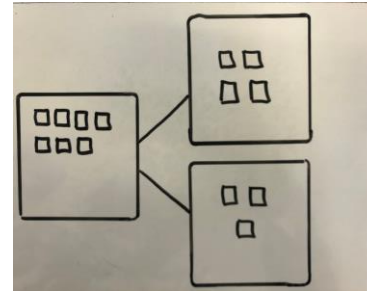
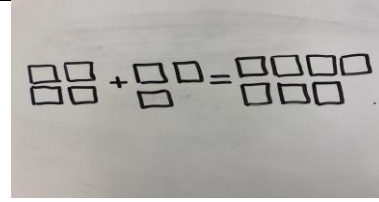
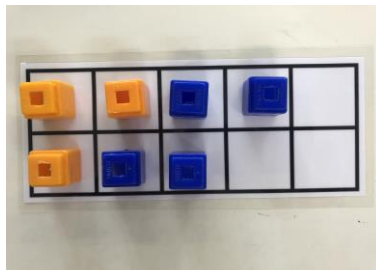
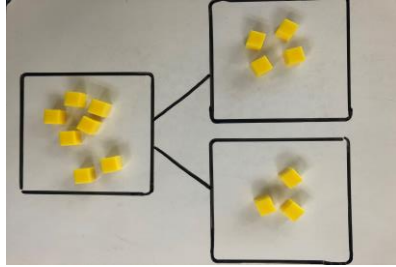
- Have a secure knowledge of number facts and a good understanding of the four calculation operations (addition, subtraction, multiplication and division)
- Make use of jottings, diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads
- Have an efficient, reliable, written method of calculation for each operation that they are able to apply with confidence when they are unable to perform a calculation mentally

Progression in Calculations

Addition

Objective and Strategies	Concrete	Pictorial	Abstract
<p><u>Year 1 Addition</u></p> <p>Given a number, identify one more and one less</p> <p>Represent and use number bonds and related subtraction facts within 20</p> <p>Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$.</p>			

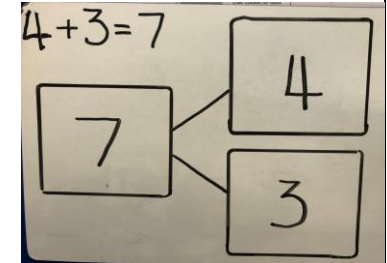
Combining two parts to make a whole: Part, part- whole model



Use pictures to add two numbers together as a group or in a bar.



$$4 + 3 = 7$$



Use the part-part whole diagram as shown above to move into the abstract.

4	3
7	

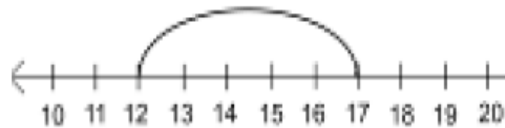
$$3 + 4 = 7$$

Starting at the bigger number and counting on



Start with the larger number on the bead string and then count on to the smaller number one by one to find the answer.

$$12 + 5 = 17$$



Start at the larger number on the number line and count on in ones or in one jump to find the answer.

$$12 + 5 = 17$$

Place the larger number in your head and count on the smaller number to find your answer.

Regrouping to make 10.

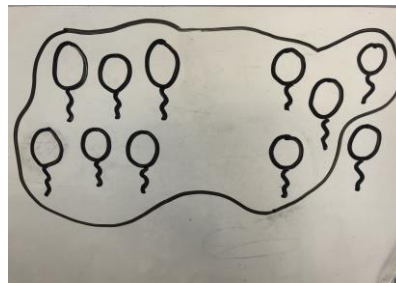
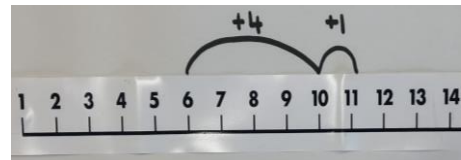
This is an essential skill for column addition later on.

Start with the bigger number and use the smaller number to make 10.

$$6 + 5 = 11$$



Use pictures or a number line. Regroup or partition the smaller number to make 10.



$$6 + 5 = 11$$

If I am at six, how many more do I need to make 10? How many more do I add on now to get to 11?

$$6 + 5 = 11$$

↓ ↓
4 1

or

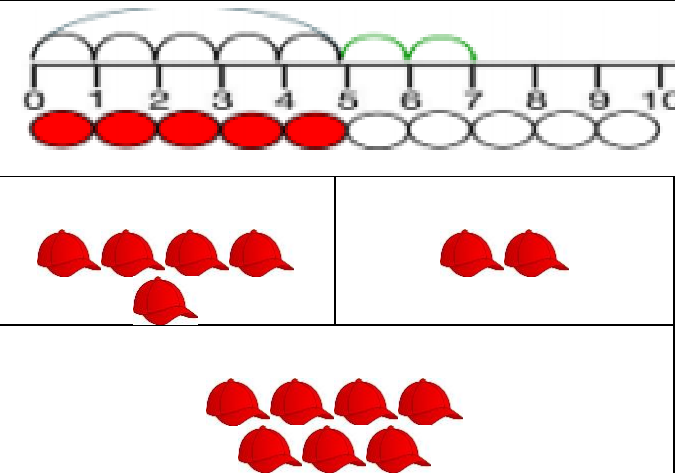
$$5 + 6 = 11$$

↓ ↓
5 1

Represent and use number bonds and related



Emphasis should be on the language: '2 more than 5 is equal to 7'

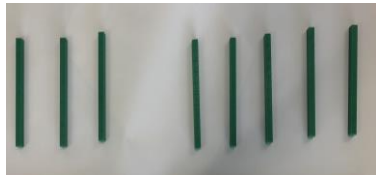
subtraction facts within 20.	2 more than 5		<p>'2 more than 5 is 7' '7 is 2 more than 5'</p> <p>Use of STEM sentences to promote this language: '_ more than _ is equal to _' '_ more than _ is _' '_ is _ more than _'</p>
Objective and Strategies	Concrete	Pictorial	Abstract

Year 2 Addition
 Use place value and number facts to solve problems recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
 Add and subtract numbers using concrete objects, pictorial representations, and **mentally**, including: TU+O, TO+T, TU+TO and O+O+O
 Show that addition of two numbers can be done in **any order (commutative)** and subtraction of one number from another cannot
 Solve problems with addition and subtraction, using concrete, pictorial and abstract representations
 Recognise and use the **inverse relationship** between addition and subtraction and use this to check calculations and solve missing number problems.

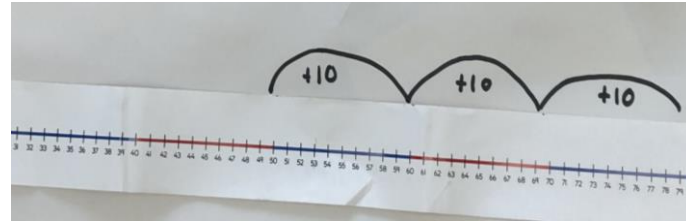
Adding multiples of 10

$$50 + 30 = 80$$

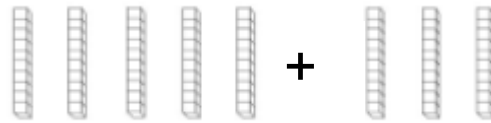
Model using Dienes and bead strings



The beads represent a numberline where we jump to 50 first, then make three jumps of ten.



Use representations for Dienes blocks.



$$30 + 50 = 80$$

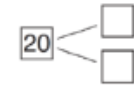
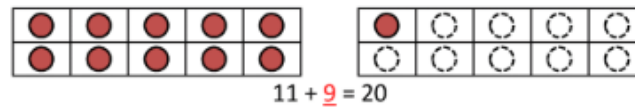
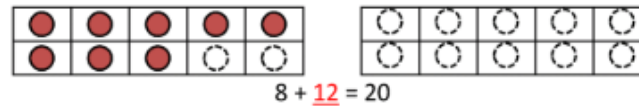
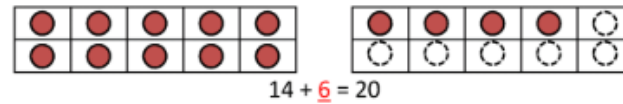
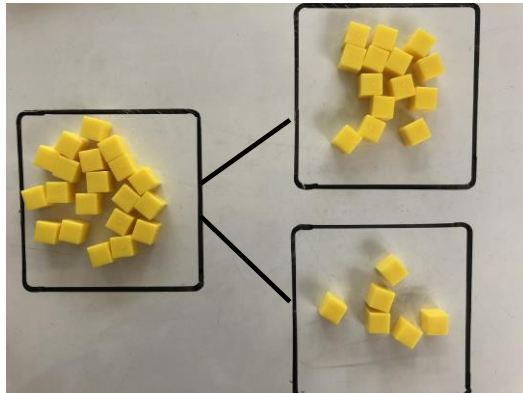
$$80 = 30 + 50$$

$$30 + \underline{\quad} = 80$$

Use known number facts.

Part, part, whole

Children explore ways of making number bonds within 20.

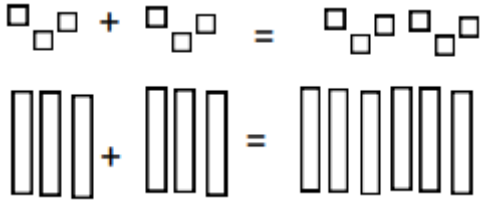
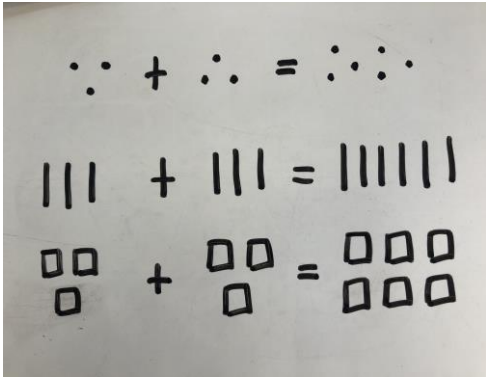


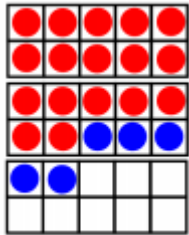
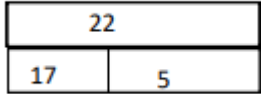


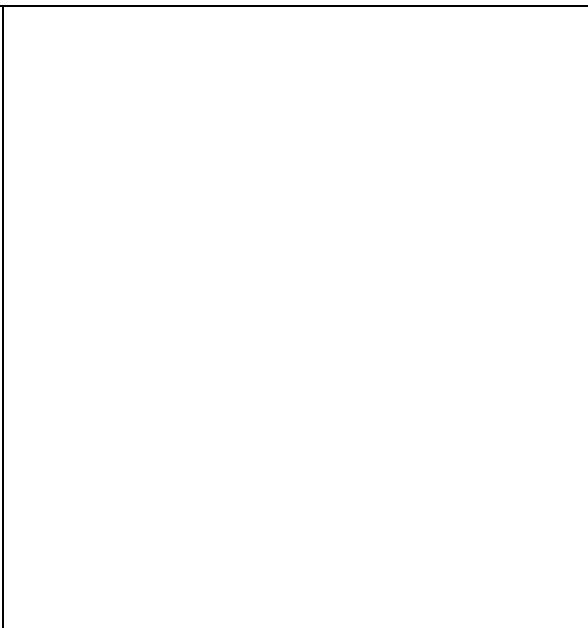
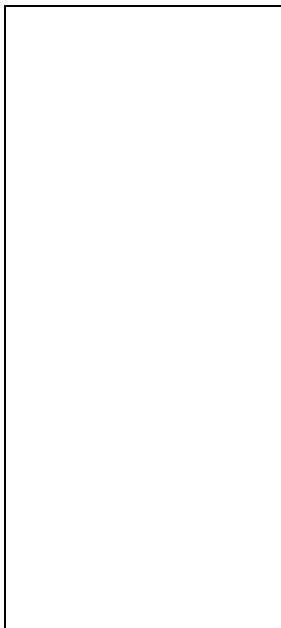
$$\square + \square = 20 \quad 20 - \square = \square$$

$$\square + \square = 20 \quad 20 - \square = \square$$

$$\square + 1 = 16 \quad 16 - 1 = \square$$

$$1 + \square = 16 \quad 16 - \square = 1$$

<p>Using known facts</p>	<p>Use Dienes to represent related additions.</p> $3 + 3 = 6$  $30 + 30 = 60$	<p>Children draw representations of hundreds, tens and ones</p>  <p>The bottom squares represent hundreds: $300 + 300 = 600$</p>	$3 + 3 = 6$ <p>leads to</p> $30 + 30 = 60$ <p>leads to</p> $300 + 300 = 600$
<p>Bar Model</p>			$4 + 3 = 7$
<p>Add a two digit number and ones</p>	<p>Use ten frame to make magic 10.</p>  <p>Children explore the pattern</p> $17 + 5 = 22$ $27 + 5 = 32$	<p>Use part, part, whole and number line to model.</p>	$17 + 5 = 22$  <p>Explore related facts:</p> $17 + 5 = 22$ $5 + 17 = 22$ $22 - 17 = 5$



$17 + 5 = 22$

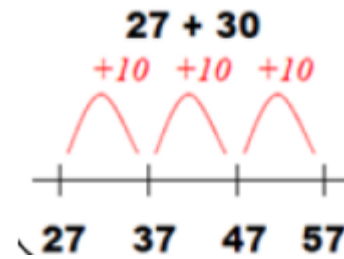
$+3 +2$

$22 - 5 = 17$

Using part whole model to add a two-digit number to another two-digit number

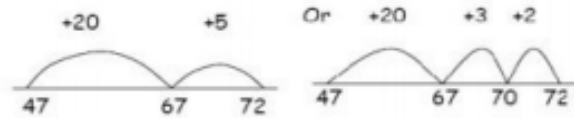


Explore that the ones digit does not change.



Use a number line and bridge ten using part, whole if necessary.

An example for solving $47 + 25$:



$$27 + 10 = 37$$

$$27 + 20 = 47$$

$$27 + \underline{\quad} = 57$$

$$25 + 47$$

$$20 + 5 \quad 40 + 7$$

$$20 + 40 = 60$$

$$5 + 7 = 12$$

$$60 + 12 = 72$$

Objective and Strategies	Concrete	Pictorial	Abstract
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Year 3 Addition

Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

Estimate the answer to a calculation and use inverse operations to check answers

Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

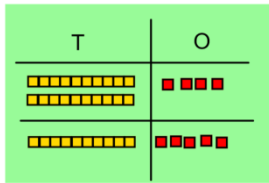
Progress to formal written methods calculations

Column method-
no regrouping

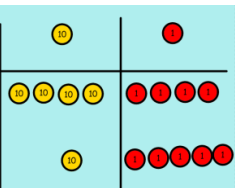
Add two or three digit numbers

Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.

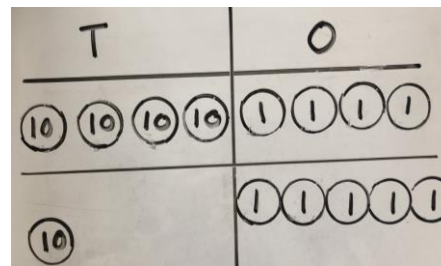
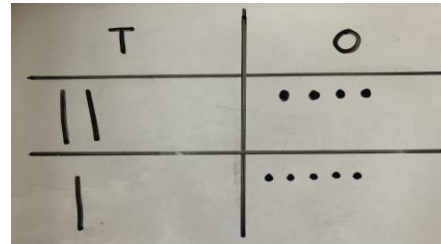
$24 + 15 =$



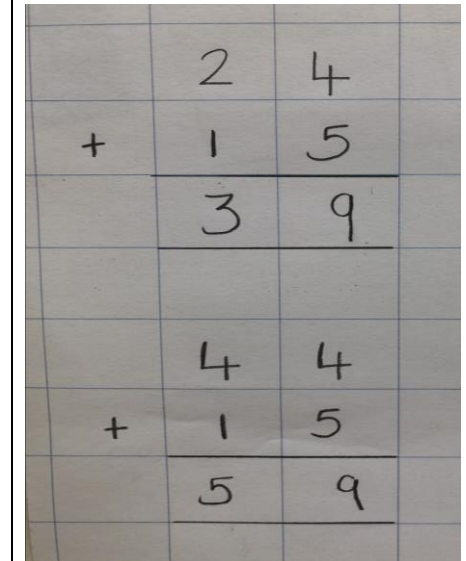
$44 + 15 = 59$

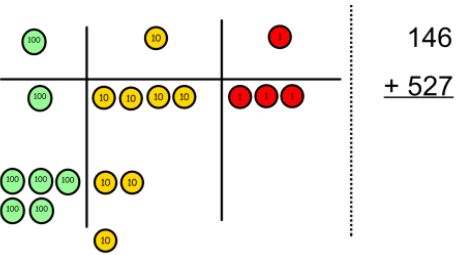
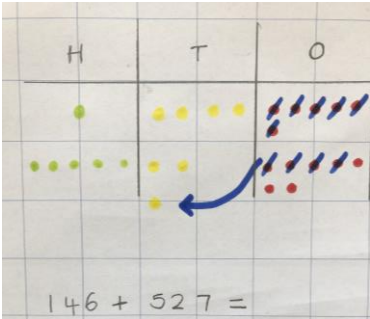


After practically using the Dienes blocks and place value counters, children can draw the counters to help them to solve additions.



Add the ones first, then the tens and then the hundreds.



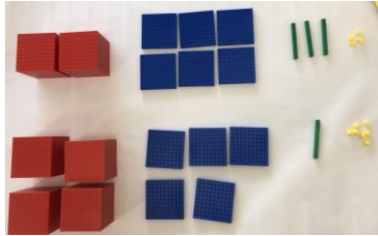
<p>Column method-regrouping</p>	<p>Make both numbers on a place value grid.</p> <p>Add up the units and exchange 10 ones for one 10. Model using PV counters.</p>  <p>Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.</p> <p>This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.</p>	<p>Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.</p>  <p>To show an exchange, they can cross out the pictorial representation of and add an extra pictorial symbol under the column to the left. It is important to put this symbol underneath the left hand column, separate to the rest of the symbols so we can see that it has been exchanged (exchanged numbers sit underneath the line in a formal written method).</p>	$\begin{array}{r} 145 \\ 537 \\ \hline 682 \\ \end{array}$
<p>Objective and Strategies</p>	<p>Concrete</p>	<p>Pictorial</p>	<p>Abstract</p>
<p><u>Year 4 Addition</u></p> <p>Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate</p> <p>Estimate and use inverse operations to check answers to a calculation</p> <p>Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why</p> <p>Solve simple measure and money problems involving fractions and decimals to two decimal places</p>			

Add numbers with up to 4 digits.

Add decimals with 2 decimal places, including money.

Children continue to use Dienes or PV counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.

$$2634 + 4517$$

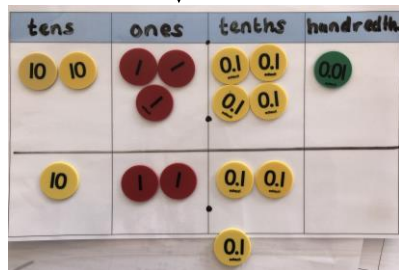


$$23.46 + 12.25$$

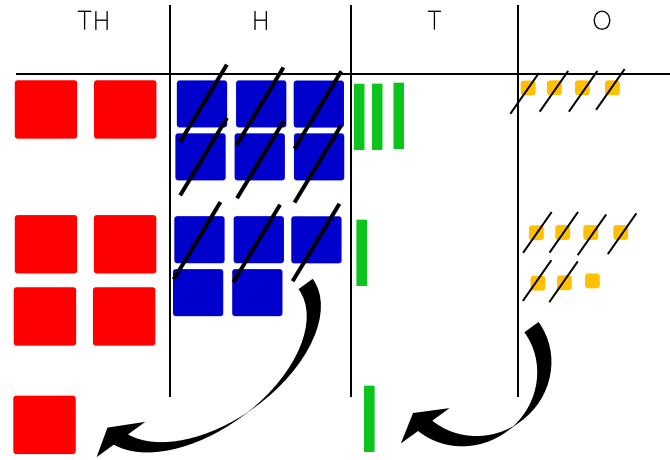
Before exchange:



After exchange:

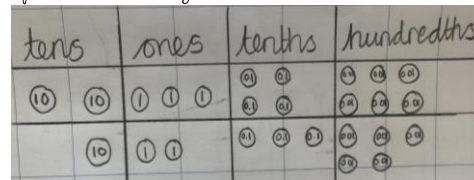


Draw representations using a PV grid.

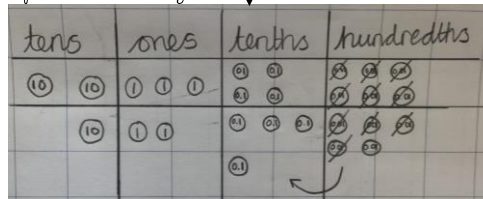


Note: Again, the exchanged symbols are carried beneath the other symbols, as represented in the abstract formal method.

Before exchange:



After exchange:



For the purpose of this document, a photograph was taken before and after the exchange. There is no need to write this out twice.

$$\begin{array}{r} 2634 \\ + 4517 \\ \hline 7251 \end{array}$$

Continue carrying hundreds as well as tens.

$$\begin{array}{r} 23.46 \\ + 12.25 \\ \hline 35.71 \end{array}$$

Relate to money and measures.

Objective and Strategies	Concrete	Pictorial	Abstract
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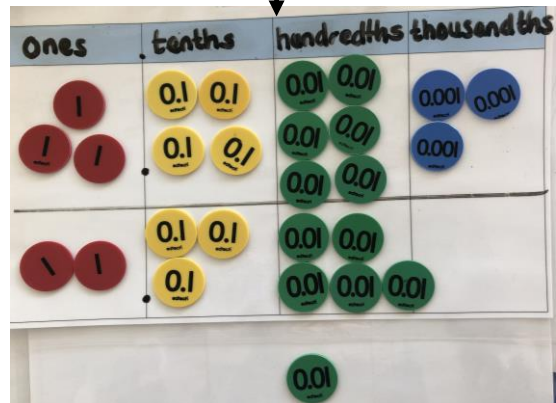
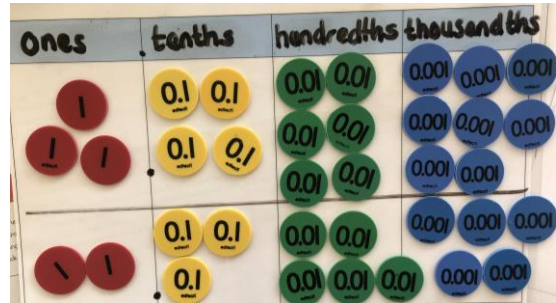
Year 5 Addition

Add and subtract whole numbers with more than 4 digits, including using formal written methods
 Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
 Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
 Solve problems involving number up to three decimal places

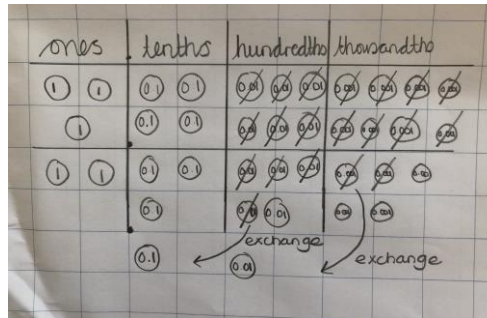
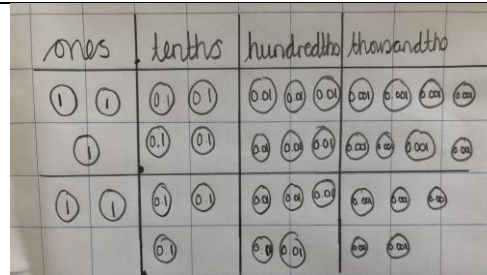
Add numbers with more than 4 digits.

Add decimals with 3 decimal places, including money.

Two exchanges taking place in the calculation



Then the hundredths would be exchanged for a tenth.



Move to the abstract method ensuring children line the decimal points correctly.

$$\begin{array}{r}
 3.468 \\
 2.355 \\
 \hline
 5.823
 \end{array}$$

Objective and Strategies	Concrete	Pictorial	Abstract
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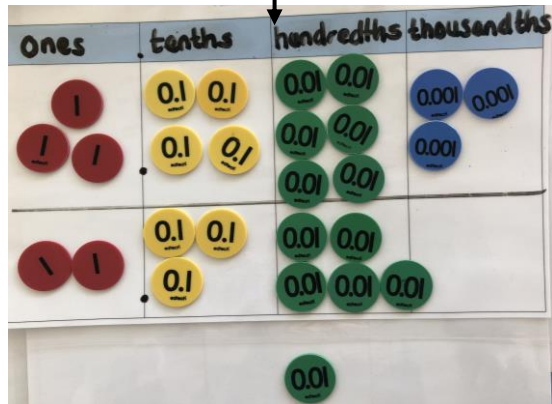
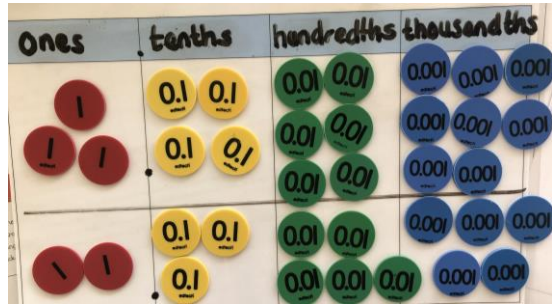
Year 6 Addition

Use their knowledge of the order of operations to carry out calculations involving the four operations

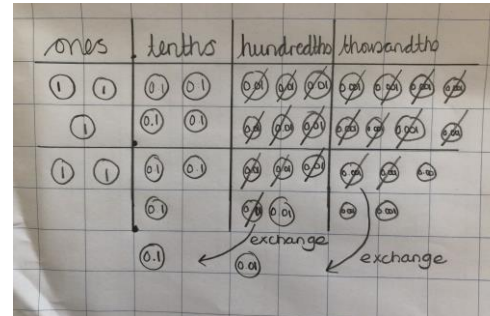
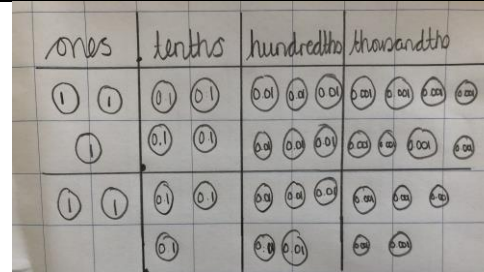
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving addition, subtraction, multiplication and division
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

Add several numbers of increasing complexity. Including adding money, measures and decimals with different numbers or decimal points.

Two exchanges taking place in the calculation



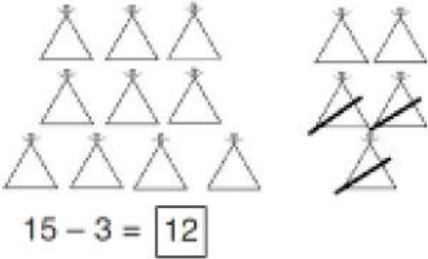
Then the hundredths would be exchanged for a tenth.



Move to the abstract method ensuring children line the decimal points correctly.

$$\begin{array}{r}
 3.468 \\
 2.355 \\
 \hline
 5.823 \\
 \hline
 \end{array}$$

Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract
<p><u>Year 1 Subtraction</u> Given a number, identify one more and one less Represent and use number bonds and related subtraction facts within 20 Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$.</p>			
<p>Taking away ones</p>	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p> <p>$15 - 3$</p>	<p>Cross out drawn objects to show what has been taken away.</p> 	<p>$15 - 3 = ?$</p> <p>$15 - ? = 12$</p> <p>$? - 3 = 12$</p>



Counting back

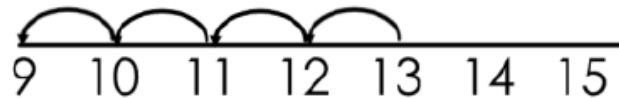
Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.

$$13 - 4$$



Use counters and move them away from the group as you take them away counting backwards as you go.

Count back on a number line or number track



Start at the bigger number and count back the smaller number showing the jumps on the number line.

Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

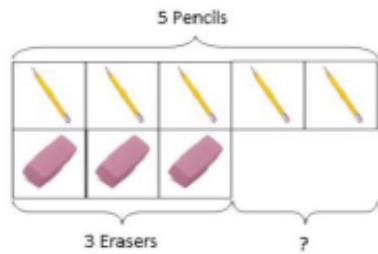
$$13 - 4 = ?$$



Find the difference

Compare amounts and objects to find the difference.

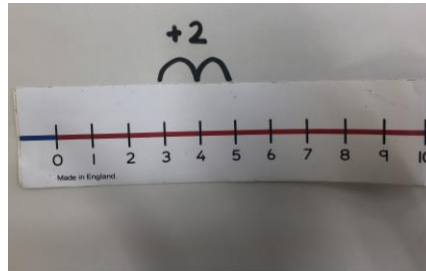
Use cubes to build towers or make bars to find the difference



Use basic bar models with items to find the difference.



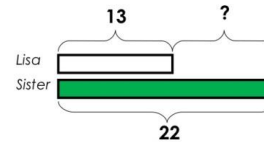
Count on to find the difference.



Draw bars to find the difference between 2

Comparison Bar Models

Lisa is 13 years old. Her sister is 22 years old.
Find the difference in age between them.



numbers.

Hannah has 3 sandwiches, Helen has 5 sandwiches. Find the difference between the number of sandwiches.

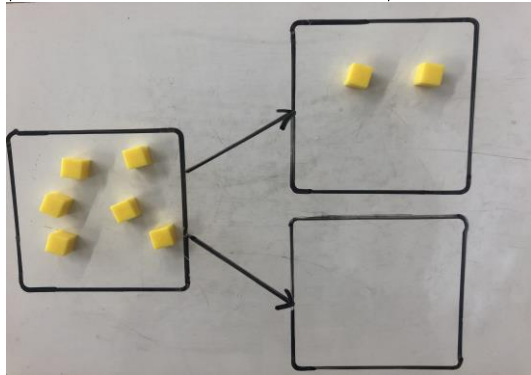
Tom works 5 days a week and Kate works 3 days a week.

Represent and use number bonds and related subtraction facts within 20.

Part, part, whole model.

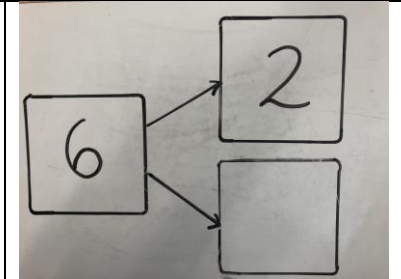
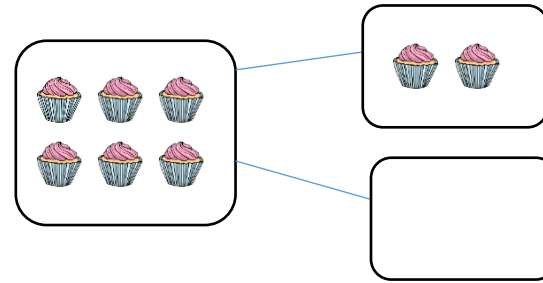
Link to addition- use the part, part, whole model to help explain the inverse between addition and subtraction.

If 10 is the whole and 6 is one of the parts. What is the other part?



$$10 - 6 = 4$$

Use a pictorial representation of objects to show the part, part, whole model.

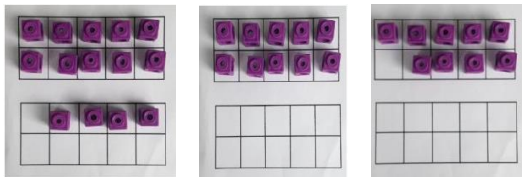


Move to using numbers within the part whole model.

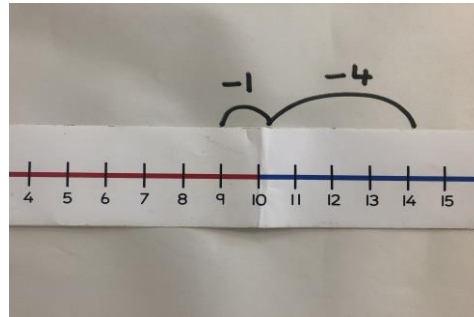
Make 10

$$14 - 5 =$$

Make 14 on the ten frame. Take away the four first to make 10 and then take away one more so you have taken away 5. You are left



with the answer of 9.


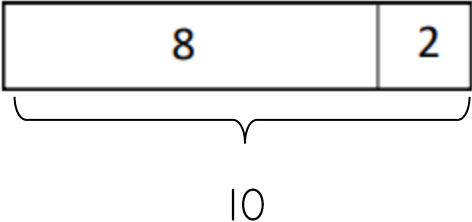
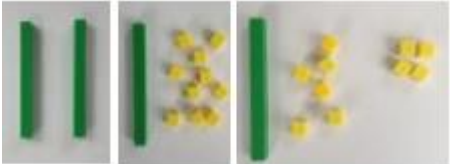
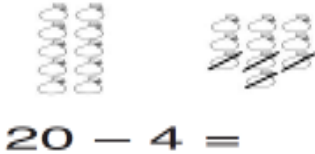



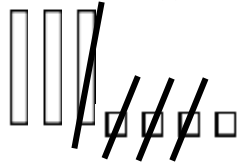
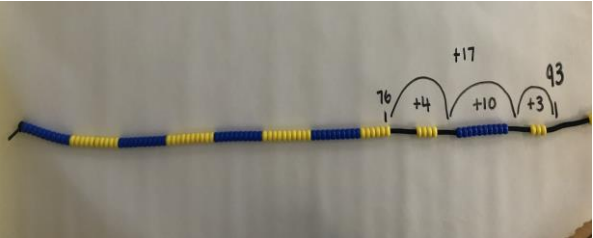
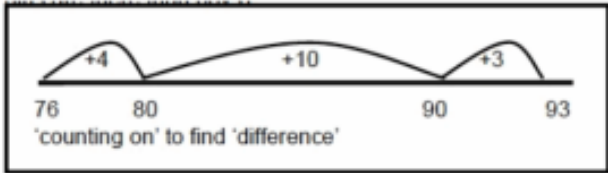

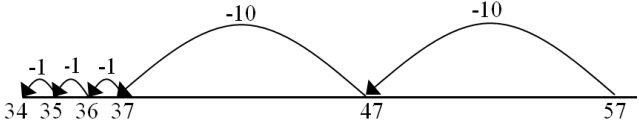
Start at 14. Take away 4 to reach 10. Then take away the remaining 1 so you have taken away 5 altogether. You have reached your answer.

$$14 - 5 =$$

How many do we take off to reach the next 10?

How many do we have left to take off?

Bar Model			$10 = 8 + 2$ $10 = 2 + 8$ $10 - 2 = 8$ $10 - 8 = 2$
Objective and Strategies	Concrete	Pictorial	Abstract
<p><u>Year 2 Subtraction</u></p> <p>Use place value and number facts to solve problems recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100</p> <p>Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: TO+O, TO+T, TO+TO and O+O+O</p> <p>Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot</p> <p>Solve problems with addition and subtraction, using concrete, pictorial and abstract representations</p> <p>Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.</p>			
Regroup a ten into ten ones	<p>Use a PV chart to show how to change a ten into ten ones, use the term 'take and make.'</p> 		$20 - 4 = 16$

<p>Partitioning to subtract without regrouping</p>	<p>$34 - 13 = 21$</p> <p>Use dienes to show how to partition the number when subtracting without regrouping.</p> 	<p>Children draw representations of Dienes and cross off.</p> 	<p>$43 - 21 = 22$</p>
<p>Making ten strategy</p> <p><i>Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</i></p>	 <p>Use a bead bar or bead strings to model counting to next ten and the rest.</p>	 <p>Use a number line to count on to the next ten and then the rest.</p>	<p>$93 - 76 = 17$</p>
<p>Counting back</p>	<p>Use a bead bar or bead strings to model counting back in tens and then ones</p>  <p>Start with 57 beads and then move 2 tens then 2 ones to the left. Start from</p>	 <p>This can progress all the way to counting back using two 2-digit numbers.</p>	<p>$57 - 23 = 34$</p>

the left hand side of the string so that the tens are easily visible.

Objective and Strategies

Concrete

Pictorial

Abstract

Year 3 Subtraction

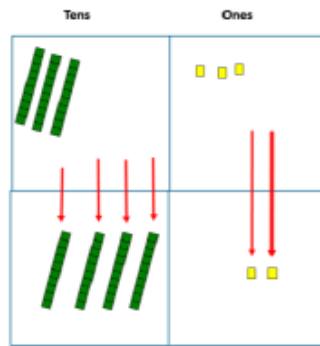
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

Estimate the answer to a calculation and use inverse operations to check answers

Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

Column method without exchange

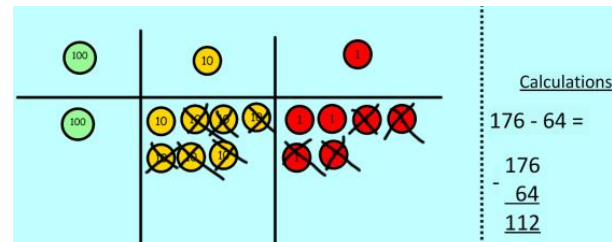
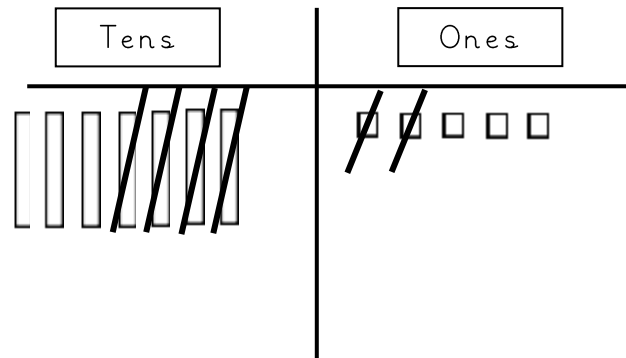
75-42



Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.

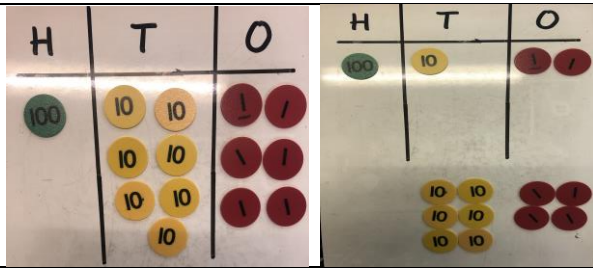
Draw the Base 10 or place value counters alongside the written calculation to help to show working.



$$\begin{array}{r} 75 \\ - 42 \\ \hline 33 \end{array}$$

This will lead to a clear written column subtraction.

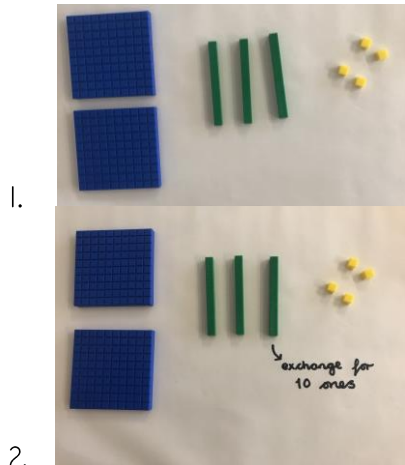
$$\begin{array}{r} 176 \\ - 64 \\ \hline 112 \end{array}$$



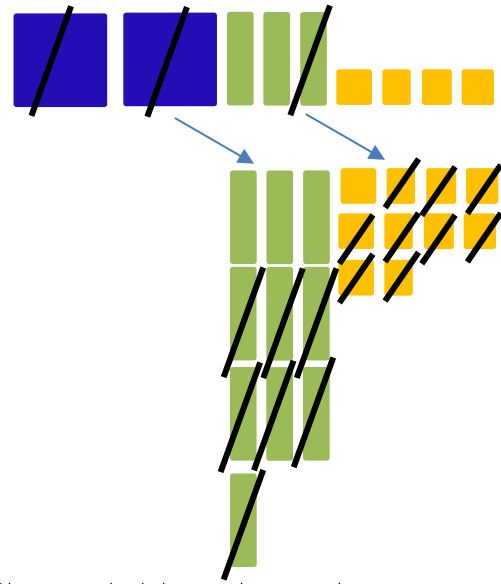
Column method with regrouping

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges in the spring term. Use the 'take and make' for exchange.

$$234 - 179 =$$



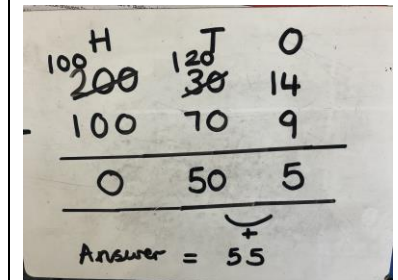
Children may draw base ten or PV counters and cross off.



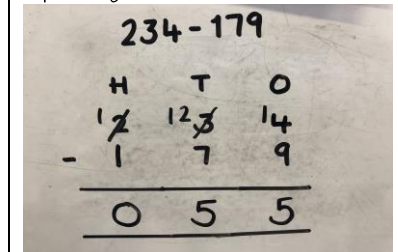
Show children how the pictorial method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

Children can start their formal written method by partitioning the number into clear place value columns and then move onto the formal method later on in the year.

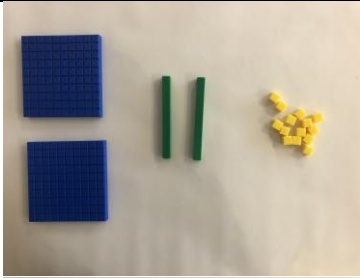
Autumn:



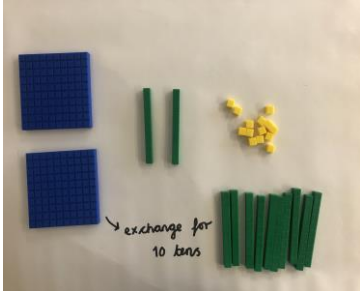
Spring:



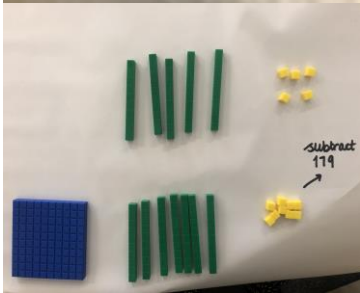
3.



4.



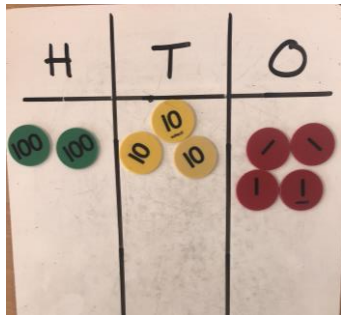
5.



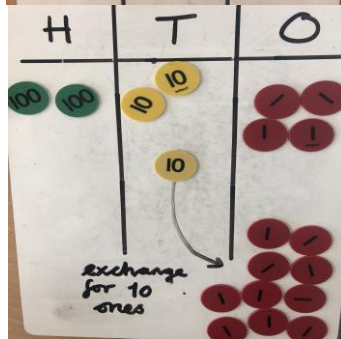
2 3 4 - 1 7 9

H	T	O
100	10 10 10	1 1 1 1
Exchange for 10 tens	Exchange for 10 ones	1 1 1 1
	10 10 10 10	1 1 1 1
	10 10 10 10	
	10 10	

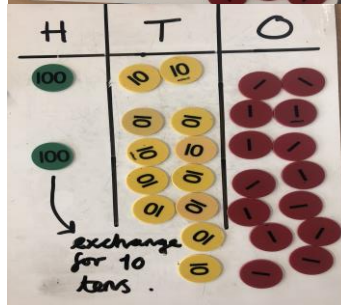
Moving onto Place Value
counters:



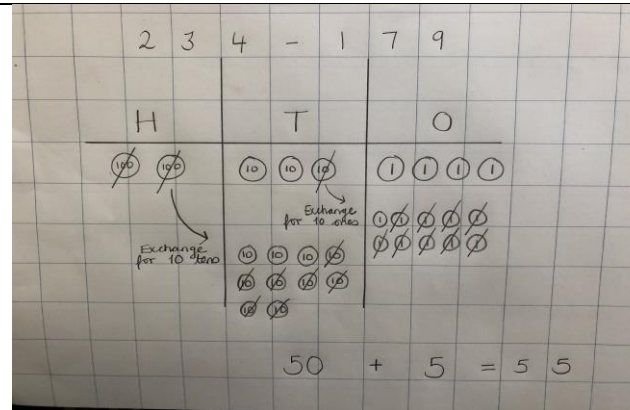
1.

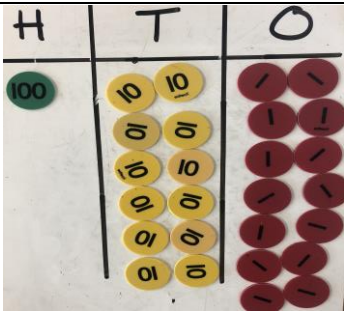
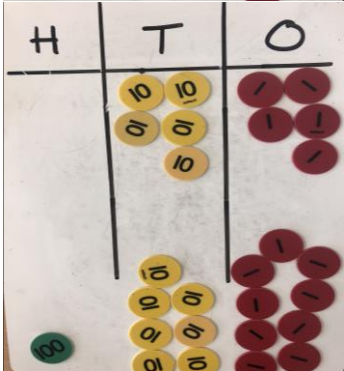


2.



3.

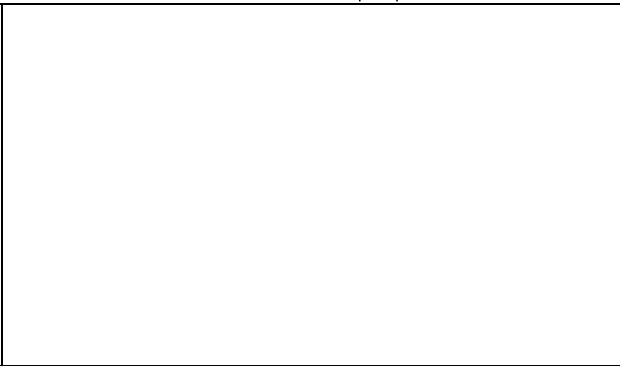
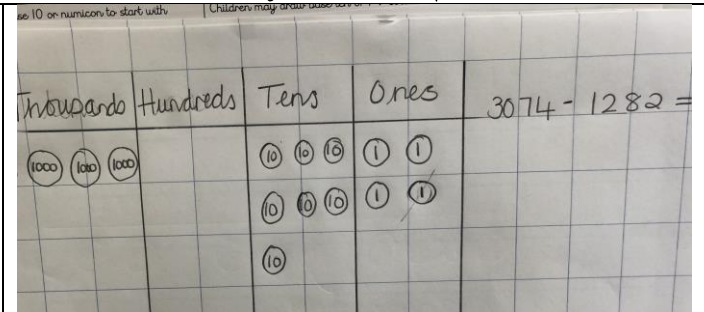


	<p>4.</p>  <p>5.</p> 		
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<u>Objective/strategy</u>	<u>Concrete</u>	<u>Pictorial</u>	<u>Abstract</u>
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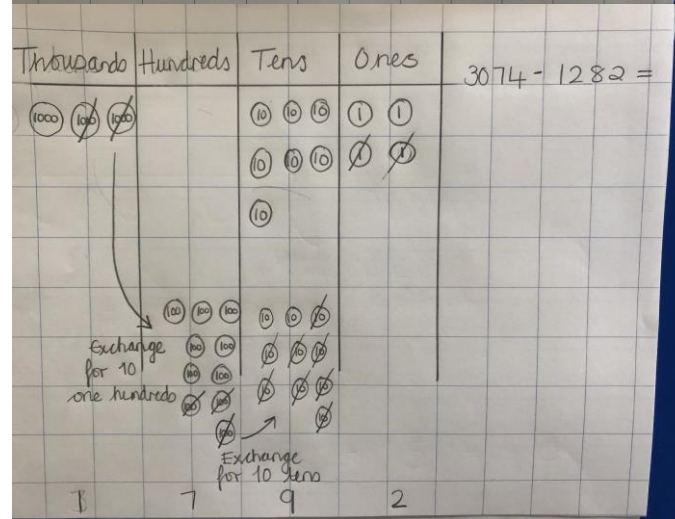
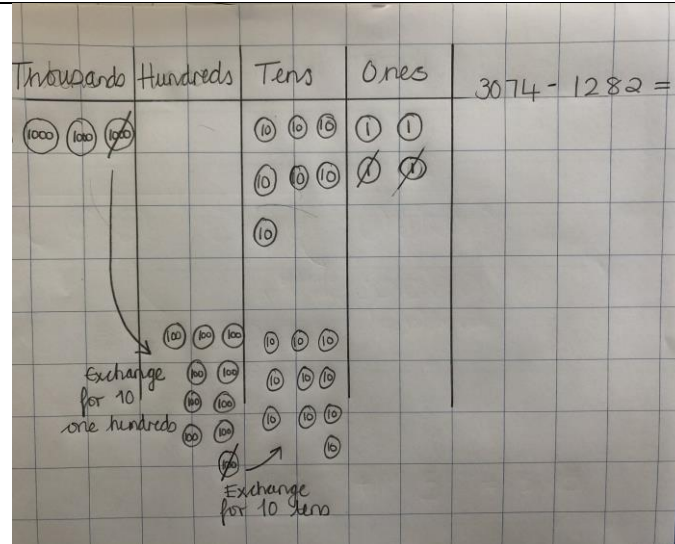
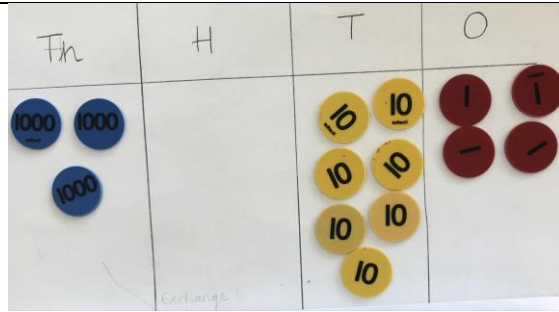
Year 4 Subtraction
 Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
 Estimate and use inverse operations to check answers to a calculation
 Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why

Subtracting tens and ones.
 Subtract with up to four digits.
 Introduce decimal subtraction through the

$$\begin{array}{r}
 3074 \\
 - 1282 \\
 \hline
 1792
 \end{array}$$

context of money.



The skills are **transferrable** over to decimal numbers. In the example below, the headings of the columns has changed and a decimal point has been added after the ones column.

Objective and Strategies	Concrete	Pictorial	Abstract
<p><u>Year 5 Subtraction</u> Add and subtract whole numbers with more than 4 digits, including using formal written methods Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why</p>			
Subtract with at least four digits, including money and measures.	As above Apply skills to problem solving, reasoning and fluency questions. Children should still be promoted to use a range of manipulatives. The greater the range of manipulatives used the greater level of mastery children will have.	As above Apply skills to problem solving, reasoning and fluency questions. Children should still be promoted to use a range of manipulatives. The greater the range of manipulatives used the greater level of mastery children will have.	As above Apply skills to problem solving, reasoning and fluency questions. Children should still be promoted to use a range of manipulatives. The greater the range of manipulatives used the greater level of mastery children will have.
Objective and Strategies	Concrete	Pictorial	Abstract
<p><u>Year 6 Subtraction</u> Use their knowledge of the order of operations to carry out calculations involving the four operations Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why Solve problems involving addition, subtraction, multiplication and division Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy</p>			
Subtract with increasingly large and more complex numbers	As above. There is a greater requirement to have a mastery of abstract	As above. There is a greater requirement to have a mastery of abstract concepts at this	Subtractions problems should include numbers with a different amount of

<p>with decimal values.</p>	<p>concepts at this point. Children should still have the fundamental knowledge provided through the CPA approach but must be able to fluently and accurately use abstract methods to solve calculations.</p>	<p>point. Children should still have the fundamental knowledge provided through the CPA approach but must be able to fluently and accurately use abstract methods to solve calculations.</p>	<p>decimal places or different amount of digits. Children should 'pad' these numbers out with a zero as a place holder, as shown below.</p> $\begin{array}{r} 80,699 \\ - 89,949 \\ \hline 60,750 \end{array}$ $\begin{array}{r} 105,419 \text{ kg} \\ - 36,080 \text{ kg} \\ \hline 69,339 \text{ kg} \end{array}$
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